MATHS - SHORT NOTES

/ / 01

unit - 01 - Matrices, system of Linear Equations

★ System of Linear equations
(1) Homogeneous -
• consistent
→ $\rho(A|B) = \rho(A)$

consistent
- unique: r = no. of unknown
- infinite: $r \neq$ no of unknown

★ Checking consistency.

① $x+y+z$ } equation.

② Augmented matrix A|B -

[coeff] | | = [ ]

A, B (n variable), B.

③ convert to echelon form

④ check $\rho(A|B) = \rho(A)$ ✓ then consistent.

★ FREE VARIABLE - Infinite solution
• Rank $\neq$ no. of unknown.
• Solve by free variable
• FV = No of unknown - Rank
• Take any variable, put as t.
• Find y, x and z.

★ system is consistent when a, b, c are in A.P
• Find Rank

★ What values of K, equations have infinite no. of solutions
① convert to echelon form
② For consistent - last element should be 0.
③ Put last eqn to 0. [| $\bar{\Theta}$] put = 0
④ Find $k = x$, $k = y$
⑤ Find infinite solution for $k = x$ or $k = y$.
⑥ Put $k = x$ in echelon form
⑦ By FV.

★ Find K for infinite and No solution
① For infinite Rank $\neq$ No of unknown
② Find a and b from echelon form
a) Both values $a = 3$, $b = 10$
→ consistent, infinite
b) $a = 3$, $b \neq 10$ b not equal
→ No solution
c) $a \neq 8$, $b \in R$ [a not equal, $b \in R$]
→ unique solution

★ HOMOGENEOUS
• Always consistent (LHS = 0)
• unique / zero solution → Rank = unknown
• Infinite $R \neq$ No of unknown

- Non trivial - infinite - dependent
- Trivial - unique, independent
- Inconsistent - No solution.

★ FIND VALUES of λ and μ
1. Find λ and μ from matrix in echelon form
2. Put $\lambda =$ __ , μ = __
3. Then find Rank.
- Infinite λ, μ put both
- No solution $\lambda =$ __ , $\mu \neq n$
- unique $\lambda \neq$, $\mu \in R$.
- Put ★ wherever $\lambda \neq$ and $\mu \neq$.

★ ~~consistent~~ Inconsistent ~~R ≠ No. of unknown~~

NON HOMOGENEOUS.

CONSISTENT $\rho(A|B) = \rho(A)$

- unique: R = unknown, $a \neq$, $b \in R$. Independent
- infinite: R ≠ unknown, a, b ✓, Dependant.

INCONSISTENT $\rho(A|B) \neq \rho(A)$.

★ checking linear dependance / independance
- v_1, v_2, v_3 ✓
1. consider $c_1 V_1 + c_2 V_2 + c_3 V_3 = 0$.
2. $c_1(\ ) + c_2(\ ) + c_3(\ ) = 0$.
3. ~~write vertical columns as Rows and make A|B matrix.~~
3. write Rows as columns and make A|B matrix → 4. echelon form.

4. Find rank
- if $R(A|B) = \rho(A)$ then consistent
  $\rho(A|B)$ Rank = unknown ~~con~~ unique
  - Independent

★ consistent only when α, β, γ are in A.P.
- $\left[\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \right]$ Row echelon rank

1. System should be consistent but last element should be 0.
   $\gamma - \beta = 0$.
2. Find α, β, γ.
3. $\beta = \frac{\gamma + \alpha}{2}$ $\quad B = \frac{C + A}{2}$ ✓

★ composite transformation which express x_1, x_2 in terms of z_1, z_2.
- $x_1 = 3y_1 + 2y_2$ | $y_1 = z_1 + 2z_2$
  $x_2 = -y_1 + 4y_2$ | $y_2 = 3z_1$

1. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightarrow (1)$
2. $y_1 = z_1 + 2z_2$
   $y_2 = 3z_1$
   $\begin{bmatrix} y_2 \\ y_1 \end{bmatrix}$ write it

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\underset{x \text{ mat}}{\begin{bmatrix} & \end{bmatrix}} \underset{y \text{ mat}}{\begin{bmatrix} & \end{bmatrix}}}_{\text{multiply}} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

3. multiply
4. $x_1 =$ ____ equation
   ____

UNIT-2-EIGEN VALUES AND EIGEN VECTORS

* characteristic equation

- 2x2 -
→ $\lambda^2 - S_1\lambda + |A|$

S_1 = sum of diagonal

- 3 x 3
→ $\lambda^3 - S_1\lambda^2 + S_2\lambda - |A|$
→ where S_2 is sum of minor element

- minor
→ $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 9 & 8 & 9 \end{vmatrix}$

Add minors.

* FINDING EIGEN VALUES.

1. characteristics eqn → Find λ
2. To find eigen vectors $(A-\lambda I)v = 0$
3. Now subtract λ from diagonal elements → Put λ.
4. Make equations
$v_1 + v_2 = 0$
$2v_1 + 3v_2 + 4v_3 = 0$

- If equations are same, then consider 1 equation.
- Crammer's Rule
→ For 3 different equations
→ Choose only 2 eqn from it

$v_1 = -v_2 = v_3$

| | | | | | |

hide v_1 and write the rest of the elements.

5. $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$

- Take common and write eigen vectors.

6. Free variable - For 2 same equation

Let $v_2 = 1$
$v_1 = 1$

$\therefore \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- eigen vector can not be 0.

* PROPERTIES OF EIGEN VECTOR

Trace (A) = sum of diagonal elements
↓
A^{-1} has eigen values $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$
↓
A^n are $\lambda_1^n, \lambda_2^n, \lambda_3^n$

|A| = Product of eigen values
↓
eigen values of KA are $K\lambda_1, K\lambda_2, K\lambda_3$
↓
eigen values of A and A^T are same

- Any non zero scalar vector / multiple of an eigen vector is also an eigen vector.

P = eigen vector
D = diagonal matrix
with eigen value

★ CAYLEY HAMILTON ~~ME~~THEORM.

① characteristics of equation
$\lambda^3 - S_1\lambda^2 + S_2\lambda - 0$ ..
Find $S_1, S_2, |A|$

② write equation.

③ By CHT, Replace λ by A
$A^3 - 6A^2 + 9A - 4I = 0 \rightarrow$ ①
Last number should have I.

④ multiply by A.
$A(A^3 - 6A^2 + 9A - 4I)$
$A^4 - 6A^3 + 9A^2 - 4A$

⑤ Then $A^4 =$ ______ From calci.

• Finding A^{-1}

① multiply by A^{-1}
$A \times A^{-1} = I$
$A^{-1}(A^3 - 6A^2 + 9A - 4I) = 0$
$A^2 - 6A + 9I - 4A^{-1} = 0$.

② $A^{-1} = \dfrac{A^2 - 6A + 9I}{4}$

③ $A^{-1} =$ ____ ✓.

★ Find a modal matrix which diagonalizes a matrix. A [ ]. ~~also find A~~.

• A [ ]

① Find eigen values and corresponding eigen vector

② write P matrix containing eigen vectors

• $P = \begin{pmatrix} 1/2 & 1 \\ 1 & 1 \end{pmatrix}$ $\left\{ \begin{array}{ll} ev_2 = & \begin{matrix} -1/2 \\ 1 \end{matrix} \\ ev_2 & \begin{matrix} 1 \\ 1 \end{matrix} \end{array} \right\}$

③ D = diagonal matrix
All eigen values in diagonal
value 5, 2; λ_1 and λ_2.

• $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$

④ Find P^{-1}.

⑤ verify PDP^{-1}.

$P \times D \times P^{-1} = [\quad]$ ✓.

★ PRINCIPLE COMPONENT ANALYSIS.

① From table write matrix A

② Mean vector A $\begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}$

$M = \dfrac{- - -}{\text{no of elements}} = M[\ ]$

③ B = A − M.

• subtract value of Mean vector from A.
Find B matrix ✓

④ S = covariance matrix $- \dfrac{1}{n-1} B \cdot B^T$

n = no. of elements.

$\dfrac{1}{n-1} \cdot B B^T$.

– $S = \dfrac{1}{n-1} [\quad]$ ✓.

SINGULAR VALUE DECOMPOSITION.

- $u = AA^T$ left singular matrix
- $V = A^TA$ right singular matrix
- For right singular matrix V

→ $AV_i = \sigma_i u_i$

$$u_i = \frac{1}{\sigma_i} AV_i$$

- left singular matrix $V_i = \frac{1}{\sigma_i} A^T \cdot u_i$

1) Find A^T and AA^T and A^TA
2) Find eigen values of A^TA or AA^T (ch eqⁿ)
3) Find σ → By CH equation.

$\sigma_1 = \sqrt{\lambda_1} = 5$

$\sigma_2 = \sqrt{\lambda_2} = 3$.

4) put values of σ_1 and σ_2 in diagonal matrix of singular values

$$\Sigma = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

Decreasing ORDER

5) To calculate eigen vector

$(AA^T - \lambda I)V = 0$. (A^TA)

6) Find eigen vectors of the matrix
7) calculate length of eigen vector.

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} t$$

length $\sqrt{(1)^2 + (1)^2} = \sqrt{\ } = \sqrt{2}$

8) write length as unitary matrix

$$u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

divide eigen vector by length

and find u_2 for second λ value.

Staring
Staring
Arr

UNIT-03 & UNIT-04 Partial Differentiation / /

- $a^x = \log a \cdot a^x$
- $e^{ax} = e^{ax} \cdot a$
- $e^{-x} = -e^{-x}$
- $\sin(ax+b) = a\cos(ax+b)$
- $\tan(ax+b) = a\sec^2(ax+b)$
- $\sec(ax+b) = a\sec(ax+b)\tan(ax+b)$
- $\sin^{-1}(ax+b) = \frac{a}{\sqrt{1-(ax+b)^2}}$
- $\cos^{-1}x = \frac{-a}{(1-x)^2}$
- $\tan^{-1}x = \frac{a}{1+(ax+b)^2}$
- $\cot^{-1}x = \frac{-a}{1+x^2}$
- $\frac{u}{v} = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$
- $\log(ax+b) = \frac{a}{ax+b}$.

* IIF derivative :
- $\frac{\sin^{-1}}{x} = 1 = \lim_{x\to 0}\frac{x}{\sin^{-1}x}$
- $\frac{\tan^{-1}x}{x} = 1 \lim_{x\to} \frac{x}{\tan^{-1}x}$
- If we get a finite value after making 0/0 form then stop the derivative.

* L'HOSPITAL RULE
- For 0/0 form → Apply n number of times
- For ∞/∞ form → apply once then convert into 0/0 form.

1. Put $x=0$
2. 0/0 form
3. Solve by L'H Rule.
4. Put $x=0$, find limit.

* Derivative and formulas
- $\sin 2x = 2\cos 2x$
- $\cos 2x = 2\sin 2x$
- $\sec 0, \tan 0, \cos \pi/2 = 0$
- $\tan \pi/2 = \infty$, $\cos 0 = 1$
- $\sin \pi/2 = 0$.
- $\sin x = \cos x$
- $\cos x = -\sin x$
- $\tan x = \sec^2 x$
- $\cot x = -\mathrm{cosec}^2 x$
- $\sec x = \sec x \cdot \tan x$
- $\mathrm{cosec}\,x = -\cot x \cdot \mathrm{cosec}\,x$
- $\ln x = \frac{1}{x}$

- $\log(0) = -\infty$
- $\log(\infty) = \infty$
- $\log(1) = 0$
- $\frac{1}{x} = -\frac{1}{x^2}$

→ $2^x = 2^x \cdot \log 2$.

- log properties

→ $\log m^n = n\log m$

→ $c\log a = \log a^c$

→ $\log_b a = \frac{1}{\log_a b}$.

$0^\infty, 1^\infty, \infty^\infty$

* Take log on both sides
- in $0 \cdot \infty$
: Reciprocal in deno → $0/0$ or ∞/∞

* Numerator condition
- Find value of p such that
  " q " is finite
→ Find derivative, LH Rule
→ Put $x = 0$ → condition
  $2+p=0$, if not then
  $L=\infty$ but which is not true.

Assume $x=0$.
then $\frac{\infty}{\infty}$ form

find p ✓ and $\frac{0}{0}$ form.

* ∞/∞ form
- $\sec x = \frac{1}{\cos x}$
- $\operatorname{cosec} = \frac{1}{\sin x}$

If ~~tan, sin~~ sec, convert them into cos x, sin x, tan x.

* $0 \times \infty$ or $\infty \times 0$.
- If ∞ occurs due to log then don't bring log to the denominator
- Reciprocal in denominator

* $\infty - \infty$ → ~~...~~
- cross multiply.

* RMVT
- continuous in $[a,b]$
- differentiable in (a,b)
- condition $f(a)=f(b)$
  then $f'(c)=0$.

take

- c is always in open interval
- No log function exists in

① $f(x) =$ ____
② condition – ✓ continuous / derivable
③ $a = x$
   $b = y$
④ $f(a)$, $f(b)$ } Put values of a, and b
⑤ $f(a) = f(b)$ ✓
   ∴ RMVT is applicable.

   $c \in (a,b)$ such that
   $f'(c) = 0$
⑥ Find $f'(c) = 0$.
⑦ replace x by c and find c
   $c \in (x,y)$.

★ LMVT
- continuous in $[a,b]$
- differentiable in (a,b)
- $f'(c) = \frac{f(b)-f(a)}{b-a}$.

- Polynomial, exp, cos x, sin x are differentiable and continuous

① Let $f(x) =$ ____
   ∴ By LMVT
   $c \in (a,b)$.
   $f'(c) = \frac{f(b)-f(a)}{b-a}$.
② find $f'(c)$
③ Put $f(a)$ and $f(b)$ and find values.

$u_y = 0 + x + 3y^2$ $\tan^{-1}(x^3+y^3)$

$u_{xx} =$

4. We get $\frac{1}{c}$

5. $c \in (a,b)$
$a < c < b$
on reciprocal,
$\frac{1}{b} < \frac{1}{c} < \frac{1}{a}$.

6. in $f(c)$, replace x by c and put equation
7. Multiply the denominator into LHS and RHS equality

★ TAYLOR SERIES

- $f(x) = 2x^3 + 7x^2 + x - 6$ in $(x-2)$

Here a is opposite.

Type-1 (1) $\cancel{x-2} \to a = 2$
$(h = x-2)$

2. $f(a+h) = f(a) + f'(a)\frac{h}{1!} + f''(a)\frac{h^2}{2!}$

3. Now find derivation
$f(x)$ } derivation upto
$f''(x)$ } the no. of degree.

4. then find $f(a)$ by putting values in $f'(x)$.

5. Put this in equation.

$f(a+h) = f(a) + f'(a)\frac{h}{1!} + f''(a)\frac{h^2}{2!}$

Type-2 Power of (x)
$h = x$, $a = 1$
$f(x) = 1 + 2(x+1)^2$ $\boxed{\text{same } h \text{ value}}$
$a + h$
take $x+1$ as t.

Q. $f(x) = x^3 - 2x^2 + 3x + 1$.
$a = 1$ $h = x-1$
$f(a+h) = f(a) + f'(a)\frac{h}{1!} + f''(a)\frac{h^2}{2!}$

$f(a) = x^2 - 2x^2 + 3x + 1$
$f'(a) = 2x - 4x + 3$.
$f''(a) = 2 - 4 + 3$
$f(1) = 3x^2 - 4x + 3 + 0$
$f'(x) = 6x - 4$
$f''(x) = 6$
$f(1)$
$f(2)$

$f(a + x - 1) = f(1) + (x-1)$
$(x-1) = a = 1$
x.
$x - 2$
$h = x$

$7(x+2) + 3(x+2)^3 + (x+2)^4 - (x+2)^5$
here $x = h$, $a = 2$
$f(x+2) = f(a) + f'(a)\frac{h}{1!} + f''(a)\frac{h^2}{2!}$

let $(x+2)$ be t.
$f(t) = 7t + 3t^3 + t^4 + t^5$
$f'(t)^3 = 3x^3$

$\sin(x^2+y^2)$ u_x, u_{xy}.
$u_x = \frac{\partial u}{\partial x} + \cos(x^2+y^2)\cdot\frac{\partial}{\partial x}$

★ $u_x = \frac{\partial u}{\partial x} + \frac{\partial^2(xy)}{\partial x}$

$u = x^2 + xy + y^3$. $\cos^2$
$\frac{\partial u}{\partial x}$, $u_{xx} =$

$x^2 + xy + y$
$u_x = 2x + y + 2y \; 0$
$u_y = 0 + x + 3y^2$

u_x ...

* PARTIAL DIFFERENTIATION
- $f(x) = \frac{\partial u}{\partial x}$ constant / variable
- $u_{xx} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$
- $u_{yy} = \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$

* SIMPLE differentiation - Homogeneous
- $u(xt, yt)$ To check homogeneous.
(1) Let $x = xt$, $y = yt$
(2) then (xt, yt)
(3) t common $\rightarrow$ homogeneous

- No trig / exp / log in example then - homogenous.

- when function has t inside Trig sin / cos etc then it is not homogeneous

$\rightarrow \sin^{-1}[t^2(x^2+y^2)]$
$\downarrow$
t^2 inside $\sin^{-1}$
which is $\neq t^2 \sin^{-1}(x^2+y^2)$
$\therefore$ Not homogenous

* $f(u) = \sin u$
is homogenous with n
- else Not homogenous equation

★ Euler's - Homogeneous
(1) single derivative
- $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu(x,y)$

(2) Double differentiation
- $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

★ Euler's - NON-HOMOGENEOUS
(1) $x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial y} = \frac{nf(u)}{f'(u)}$

(2) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)\,[g'(u)-1]$

where $g(u) = \frac{nf(u)}{f'(u)}$

(1) Find homogenous degree
(2) $\therefore$ By Eulers $\rightarrow$ Non homogeneous
$\rightarrow$ write statement $\rightarrow$ (1)
(3) find $g(u) = \frac{nf(u)}{f'(u)} = \boxed{}$
(4) Then find $g'(u)$
(5) Put in statement (1).

• composite function
• If $u = f(\ldots)$
If $u \to x, y$

COMPOSITE FUNCTION

1) $z = 2x + y \to u, v$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$$

2) $u \to P, Q, R \to x, y, z$

(1) (2) (3) (4)

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial P}\cdot\frac{\partial P}{\partial x} + \frac{\partial u}{\partial Q}\cdot\frac{\partial Q}{\partial x} + \frac{\partial u}{\partial R}\cdot\frac{\partial R}{\partial x}$$

$$\frac{\partial u}{\partial P} \quad \frac{\partial u}{\partial Q} \quad \frac{\partial u}{\partial R}$$

★ Remember Formulae

• $\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right) \cdot n = 1/2$ | $\frac{x^2+y^2}{x^2+2xy}$, $n = 0$

• $\left(\frac{x^3+y^3}{x+y}\right) = n = 2$

$\sec^2 x = 1 + \tan x$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

• $\cos 2\theta = \cos^2\theta - \sin^2\theta$

• $\cos 2\theta = 2\cos^2\theta - 1$

• $\cos 2\theta = 1 - 2\sin^2\theta$

• $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

UNIT 5: Application of PD.

★ ERROR

(1) given $A = \lambda ab$.
minor with given errors.
major

$\frac{da}{a} \times 100 = 2$, $\frac{db}{b} \times 100 = 3$.

(2) claim: $\frac{dA}{A} \times 100$ (Area).
(to find)

(3) $A = \lambda ab$ Value from area.

(4) take log when constants are multiplied.

$\log A = \log a + \log b$.

$\therefore \frac{dA}{A} = \frac{da}{a} + \frac{db}{b}$.

(5) Multiply with 100

$\frac{dA}{A} \times 100 = \frac{da}{a} \times 100 + \frac{db}{b} \times 100$

$\frac{dA}{A} = 2 + 3 = 5$.

$\therefore$ Error $= 5\%$.

★ NOTE

(1) when constants are multiplied, take log.

(2) when constants are in fraction or separated, then - do derivative.

★ RESISTOR

• $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

(1) $\frac{dR}{R} = 100\frac{dR_1}{R_1} + 100\frac{dR_2}{R_2} + 100\frac{dR_3}{R}$

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- Differentiate – Resistance, Focal length
- Log – V=IR and Area.

② Differentiate –

$-\frac{dr}{r^2} = -\frac{dr}{r_1^2} + \frac{dr_2}{r_2^2} + \left(-\frac{dr}{r_3^2}\right)$

③ Multiply with 100 and (–1)

$\frac{dr}{r^2} \times 100 = -\frac{dr \times 100}{r_1} + \frac{dr_2 \times 100}{r_2^2} - \frac{dr_3 \times 100}{r_3^2}$

④ write $\frac{dr}{r^2}$ as $\frac{dr}{r} \cdot \frac{1}{r}$.

$\frac{dr}{r^2} \cdot 100 = \frac{dr \cdot 100}{r_1}\left(\frac{1}{r_1}\right) + \frac{dr \cdot 100}{r_2}\left(\frac{1}{r_2}\right)$

we know these values $+ \frac{dr \cdot 100}{r_3}\left(\frac{1}{r_3}\right)$

⑤ $\frac{dr \cdot 100}{r_2} = \frac{1.2}{r_1} + \frac{1.2}{r_2} + \frac{1.2}{r_3}$

⑥ Take log in LHS and common.

$\log \frac{dr \cdot 100}{r_2} = 1.2\underbrace{\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)}_{1/r}$

⑦ $\log \frac{dr \cdot 100}{\cancel{r_2}} = \frac{1.2}{\cancel{r}}$

$\log dr = \frac{r \times \cancel{r} \times 1.2}{\cancel{r} \times 100} =$

$\log dr = 1.2r =$

% error is 1.2.

★ FOCAL LENGTH

$\frac{100\, dv}{v} = \frac{100\, du}{u}$

$-\frac{\partial f}{f^2} = -\frac{vd \times 100}{v^2} - \frac{du \times 100}{u^2}$

★ $I = E/R$ by differentiating log

① given $I = 15$ amp

$E = 100V$.

$dI = \frac{}{} \; dE = 0.5$

② Claim $\frac{dR}{R}$

③ $I = \frac{E}{R} \Rightarrow R = \frac{E}{I}$

④ Take log –

$\log R = \log E - \log I$.

⑤ Differentiate

$\frac{dR}{R} = \frac{dE}{E} - \frac{dI}{I}$

⑥ Multiply by 100

$\log 100\frac{dR}{R} = 100 \times \frac{0.5}{100} - \frac{100 \cdot 0.1}{15}$

$\log \frac{dR}{R} = 100\left(\frac{0.5}{100} - \frac{0.1}{15}\right)$

$\log \frac{Dr}{r} = 0.166\ \%$

★ MAXIMA / MINIMA

① $rt - s^2$ +ve, r +ve } minima

② $rt - s^2$ +ve, r –ve } maxima.

③ zero $rt - s^2$ → Test fail.

★ Finding Maxima / Minima

① $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ →.

② Find x and y

③ Point is (x, y).

④ $r = \partial^2 f$ → differentiate ...

Derivative of y w.r.t. x

(5) $s = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(\ )$

y only

(6) $t = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}(2y) = 2$

(7) $\therefore rt - s^2 \Big|_{(x,y)} = \_\_\_ > 0$

$r \Big|_{(x,y)} = \_\_\_ > 0$

$\therefore$ ~~+ +~~ $+ -$ so maxima

(8) (x,y) is point of maxima and maximum value is

$f(x,y) = \_\_\_\_$

(1) $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$

Find u_x, u_y, v_x, v_y.

(2) $J = \begin{vmatrix} & \end{vmatrix}$

(3) Make $J = 0$.

(4) Find relation.

*

(1) Let $f_1(x,y,u,v) = 0$

$f_2(x,y,u,v) = 0$

$J = \frac{\partial(u,v)}{\partial(x,y)} (-1)^2 \frac{\frac{\partial(f_1,f_2)}{\partial(x,y)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}}$

(2) Let f_1, f_2, f_3

$J = \frac{\partial(x,y)}{\partial(u,v)} (-1)^3 \frac{\frac{\partial(f_1,f_2,f_3)}{\partial(x,y,z)}}{\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}}$

* JACOBIAN'S DETERMINANT

- $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$

when $u = \_\_$, $v = \_\_$ given

* Finding J'

- $J' = \frac{1}{J}$

- If $J = \frac{\partial(u,v)}{\partial(x,y)}$, $J' = \frac{\partial(x,y)}{\partial(u,v)}$

$JJ' = 1$

(1) f_1 ✓, f_2 ✓ $\quad \frac{N}{D}$

(2) Find $N = \frac{\partial(f_1,f_2)}{\partial(x,y)} = \begin{vmatrix} f_{1x} & f_{2x} \\ f_{2x} & f_{2y} \end{vmatrix}$

Find D ✓.

*

- $J = 0$, then dependent

- where $uv + xy$, u the simplifies } function given but without u and v specified.

- $\frac{x+y}{x-y}$ = x ~~wrt~~ x = $\frac{-2y}{(x-y)^2}$

  opposite variable in numerator.

- $\sin(A+B) = \sin A \cos B + \sin B \cos A$
- $\sin(A-B) = \sin A \cos B - \sin B \cos A$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- d. $\frac{d}{dx}$ of $y\sqrt{1-x^2}$ = $\frac{y \cdot 2x}{2\sqrt{1-x^2}}$

- $x \cdot y$ = wrt x = y
- $y^2 x$ = wrt y = $2yx$
- $\frac{y}{x}$ wrt x = $\frac{-1y}{x^2}$

★

- Square → $\times$ 1 → Root →
  Reciprocal → -1 again
  SMRRM